Does a Central Clearing Counterparty Reduce Counterparty Risk?

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Preliminary

First version: February 12, 2009

This version:[†]   March 9, 2009

Abstract

We show whether adding a central clearing counterparty (CCP) for a particular asset class, such as credit derivatives, improves the efficiency of counterparty risk mitigation and collateral demands, relative to bilateral netting between pairs of dealers. We show that, for plausible cases, adding a CCP for one class of derivatives such as credit default swaps (CDS) can actually reduce netting efficiency and thereby lead to an increase in collateral demands and average exposure to counterparty default. We also show that whenever it is efficient to introduce a central clearing counterparty, it cannot be efficient to introduce more than one CCP for the same class of derivatives.

A central clearing counterparty (CCP) stands between over-the-counter derivatives counterparties, mainly for the purpose of insulating counterparties from default losses. We show whether adding a central clearing counterparty (CCP) for a particular asset class, such as credit derivatives, reduces counterparty exposures and collateral demands. For plausible cases, adding a new CCP for one class of derivatives, such as credit default swaps (CDS), can reduce netting efficiency and thereby lead to an increase in collateral demands and average exposure to counterparty default.

Regarding the debate over whether European and American dealers should have separate CCPs,[1] we show that whenever a single CCP reduces average counterparty exposures, relative to the current standard of bilateral netting, it is never efficient to introduce another central clearing counterparty for the same class of derivatives. Of the current proposed and approved CDS central clearing counterparties, two are based in the United States, while five proposals are Europe-based.[2]

Our results are based on a simple abstract model, but help to clarify an important tradeoff between two types of netting opportunities, namely bilateral netting between pairs of dealers across different underlying assets, versus multilateral netting among many dealers across a single class of underlying assets, such as credit default swaps (CDS). The introduction of a CCP for a particular class such as credit derivatives is only effective if the opportunity to get multilateral netting in that asset class dominates the resulting loss in bilateral netting opportunities across other asset classes, including OTC deriatives for equities, interest rates, commodities, and foreign exchange.

For instance, if Dealer A is exposed to Dealer B by \$100 million on CDS, while at the same time Dealer B is exposed to Dealer A by \$150 million on interest-rate swaps, then the introduction of central clearing dedicated to CDS increases the maximum loss between these two dealers, before collateral and after netting, from \$50 million to \$150 million. In addition to any collateral posted by Dealer A to the CCP for CDS, Dealer A would need to post a significant amount of additional collateral to Dealer B. Collateral is a scarce resource, especially in a credit crisis. The introduction of a CCP for CDS can nevertheless be effective when there are extensive opportunities for multi-lateral netting. For example, if Dealer A is exposed by \$100 million to Dealer B through a CDS, while Dealer B is exposed to Dealer C for \$100 million on the same CDS, and Dealer C is simultaneously exposed to Dealer A for the same amount on the same

[1]See, for example, "New Move on CDS Clearer for Europe", by Nikki Tait, *Financial Times*, Friday February 13, 2009.

[2]U.S.-based proposals are those of ICE, already approved, and the Chicago Mercantile Exchange. See "ICE Starts Credit-Default Swap Clearing in Bid for \$400 Million," by Matthew Leising, Bloomberg, March 9, 2009. The Europe-based proposals include those of NYSE-Liffe-LCH.Clearnet, Eurex, ICE Trust Europe (an arm of ICE dedicated to Europe-based CDS clearing), LCH.Clearnet SA (a French subsidiary of LCH, dedicated to Eurozone CDS clearing), and the CME Group ("CME in Talks to Launch Clearing Service in London," Jeremy Grant, FT.com, February 27, 2009).

CDS, then a CCP eliminates this unnecessary circle of exposures. The introduction of a CCP involves an important tradeoff.

Naturally, our results show that introducing a CCP for a particular set of derivatives is helpful if and only if the number of dealers is sufficiently large relative to the exposure on derivatives that continue to be bilaterally netted. For plausible parameters, we show that it is far from obvious that this condition is currently met for the central clearing of credit default swaps. Any benefits of a central clearing counterparty dedicated to credit derivatives has been significantly reduced through the recent aggressive use of compression trades, which has lowered exposures in the CDS market to roughly half of their mid-2008 levels. Proposals by European regulators to have more than one CCP for credit default swaps, including one dedicated to European dealers, could further reduce the netting opportunities of a CCP. We provide numerical examples that give an idea of the impact of this proposal on expected counterparty exposures.

For example, working with eight prominent CDS dealers, the ICE has received regulatory approval for a central clearing counterparty dedicated to credit default swaps. Our results suggest that clearing CDS through a dedicated central clearing counterparty improves netting efficiency for eight similarly sized dealers if and only if the fraction of a typical dealer's expected exposure attributable to CDS is at least 88% of the total expected exposure of remaining bilaterally netted classes of derivatives. For 26 dealers clearing through the same CCP, the cutoff level for this exposure ratio of CDS to other derivatives classes drops to about 41.7%.

Our results show that a single central clearing counterparty that clears both credit derivatives and interest rate swaps is likely to offer significant reductions in expected counterparty exposures, even for a relatively small number of dealers.

We do not attempt to accurately measure the current benefits of introducing a CCP for credit derivatives. Our model is too simple to give reliable magnitudes. Moreover, as we will explain, some important aspects of systemic risk are not captured by the criterion that we use for judging the exposure reduction offered by a CCP. In particular, we do not consider the extent to which CCPs can mitigate the likelihood and severity of knock-on defaults that propagate through the market at the failure of a large counterparty. This would depend in part on the collateral and guarantees that dealers offer to a CCP. Nevertheless, our results make it clear that regulators and dealers should carefully consider the tradeoffs involved in carving off a particular class of derivatives for clearing. This makes sense only if the class of derivatives to be cleared is big enough and if the subset of dealers clearing through the same central clearing counterparty is large enough. So far, proposals for CDS clearing have not made this case effectively. Proposals for a number of distinct new CCPs dedicated to credit default swaps raise a particular concern.

Netting Efficiency in an OTC Market

We consider N counterparties, whom we shall call "dealers," whose over-the-counter (OTC) exposures to each other are of concern. These dealers may also have exposures with non-dealer counterparties through their prime brokerage and other businesses, but we are most interested in exposures between dealers, because dealers tend to present the greatest exposures to each other, are relatively important from the viewpoint of systemic risk, and are more likely than non-dealers to have access to CCPs. In applications, our model can be applied to any set of entities, including dealers and large hedge funds, that are considering clearing a particular class of derivatives.

We consider the opportunity for dealers to novate (that is, to re-assign) some OTC exposures to a central clearing counterparty (CCP). For example, if dealers i and j have a CDS position by which i buys protection from j, then both i and j can novate to a CCP, who is then the seller of protection to i and the buyer of protection from j. Novation to a CCP is sometimes called "clearing," although the term "clearing" is often used in other contexts.[3]

We allow for K classes of derivatives. These classes could be defined, as traditionally, by the underlying asset classes, such as credit, interest rates, foreign exchange, commodities, and equities. One can also construct derivatives classes by grouping more than one underlying asset type.

For dealers i and j, let X_{ij}^k be the total exposure (when positive) of i to j of all positions in some derivatives class k, before considering netting and collateral. By definition,

$$X_{ij}^k = -X_{ji}^k. \tag{1}$$

Before setting up a CCP, this exposure X_{ij}^k is uncertain because the level of gross current credit exposure (before collateral and netting) that will exist on a typical future day has yet to be determined. The uncertainty in X_{ij}^k also includes the risk associated with marks to market that will occur before additional collateral can be requested and received. If dealer j defaults and $X_{ij}^k > 0$, then dealer i loses X_{ij}^k from positions in asset class k, before considering the benefits of netting across asset classes, collateral, and default recovery. Default recoveries are typically those of a senior unsecured creditor.[4]

For now, we suppose that all exposures (X_{ij}^k) are of the same variance and are independent across asset classes and pairs of dealers, excluding the obvious case represented by (1). We later relax the independence assumption and the equal-variance assumption. For simplicity, we assume symmetry in the distributions of exposures across all

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[3]See Bliss and Steigerwald (2006), Pirrong (2006), and Pirrong (2009) for discussions of CCPs in the context of the over-the-counter derivatives market.

[4]Bliss and Kaufman (2006) provide an analysis of the legal implications of settlement of OTC exposures at default.

pairs of dealers. This implies in particular that $E(X_{ij}^k) = 0$. Although we do not relax the symmetry assumption in this paper, results for the general case of asymmetric dealer exposures can be obtained analytically or by Monte Carlo simulation. With N dealers and K asset classes, there are $K \times N \times (N-1)/2$ exposure distributions to be specified. Symmetry allows a dramatic reduction in the dimension of the problem.

A reasonable measure of the netting efficiency offered by a market structure is the average, across dealers, of total expected counterparty exposures, after netting, but before collateral. Before considering the introduction of a CCP, the netting efficiency is

$$\phi_{N,K} = (N-1)E\left[\max\left(\sum_{k=1}^{K} X_{ij}^k, 0\right)\right],$$

where we have used symmetry by fixing attention on a particular dealer i. Assuming normality, we have

$$\phi_{N,K} = (N-1)\sigma\sqrt{\frac{K}{2\pi}},$$

where σ is the standard deviation of X_{ij}^k.

For given collateralization standards, the risk of loss caused by a counterparty default is typically increasing in average expected exposure. (Under normality and symmetry, essentially any reasonable risk measure is increasing in expected exposure.) Risk of loss from counterparty default is a first-order consideration for systemic risk analysis.

Going beyond counterparty default risk, as expected exposures go up, the expected amount of collateral that must be supplied goes up. Collateral use is expensive. In an OTC market without a CCP, whatever collateral is supplied by one counterparty is received by another, so the net use of collateral is always zero. The need to supply collateral is nevertheless onerous, for several reasons. First, some individual counterparties on a given day will have more collateral supplied to others than others have supplied to them. The net drain on the assets that could be supplied as collateral is costly, because of the opportunity cost to use that collateral for secured borrowing, as a cash management buffer, or for purposes of securities lending as a rent-making business. Second, there is a question of the timing of collateral settlement. One must often supply collateral to a particular counterparty on a given day before collateral is received from another counterparty. If this were not the case, for instance, there would be no specials in treasury repo markets. This sort of frictional demand for collateral, analogous to the demand for money that arises from a limited velocity of circulation of money, is considered by Duffie, Gârleanu and Pedersen (2002). So long as the average cost of supplying collateral to others is larger, on average, than the average benefit of receiving collateral from others, a market with poorer netting efficiency is also a mar-

ket with higher net cost of collateral use. For a simple illustration, if the amount of collateral to be supplied is on average some multiple U of exposure, and if the average benefit b per unit of collateral value received is less than the average cost c per unit of collateral value supplied, then the average net expected cost to a dealer of collateral usage arising from counterparty exposure is $(c - b)U\phi_{N,K}$, where $\phi_{N,K}$ is the average total expected exposure measure defined above.

Although average expected exposure, after netting and before collateral, is a reasonable measure of a market's netting efficiency and is closely related to systemic risk, this measure misses some important aspects of systemic risk. Most importantly, this measure does not consider the role of correlation of defaults across dealers. In particular, as opposed to the joint solvency analysis of Eisenberg and Noe (2001), our netting efficiency measure does not consider the implications of jointly determined defaults in a network of dealers. For example, the likelihood that dealer i cannot cover its payments to j plays a causal role in determining the likelihood that dealer j cannot cover its payments to dealer m, and so on. Adding a CCP to a dealer network could in principle increase or decrease the potential for jointly determined defaults, depending on the capitalization of the CCP and of the dealers, and on the collateralization standards used by dealers and central clearing counterparties. (IOSCO has provided one set of standards for the operational risk and capitalization of CCPs.) In addition to the capital that it holds, a CCP is typically backed by dealer guarantees. In a given scenario, the quality of those guarantees depends on the extent of any dealer losses that may trigger a call on those guarantees. A full analysis of the implications of a CCP for the joint solvency of a network of dealers is beyond the scope of our research.

The assumption of normality clearly does not apply well for the exposures of many individual derivatives positions, such as individual CDS contracts, which have heavily skewed market values due to jump-to-default risk. Aggregating within the class of all CDS, however, may result in a net exposure of one dealer to another within that class that is substantially less skewed, given the diversification across underlying names and the effect of aggregating across long and short positions. Two dealers running large active matched-book CDS prime brokerage businesses may have almost no skew in the distributions of their exposures to each other.

Netting Efficiency with a CCP

We consider the implications of a CCP for one class of derivatives, say Class K. Taking the previously described setting, suppose that all positions in Class K are novated to

the same CCP. The expected exposure of dealer i to this CCP is then

$$\gamma_N = E\left[\max\left(\sum_{j\neq i} X_{ij}^K, 0\right)\right] = \sqrt{\frac{N-1}{2\pi}}\sigma. \tag{2}$$

The expected exposure of dealer i with the other $N-1$ dealers for the remaining $K-1$ classes of derivatives is $\phi_{N,K-1}$. Thus, with a CCP for one class of derivatives, the average dealer expected exposure is

$$\phi_{N,K-1} + \gamma_N. \tag{3}$$

Introducing a CCP for this single class of derivatives therefore improves netting efficiency if any only if

$$\gamma_N + \phi_{N,K-1} < \phi_{N,K} \Leftrightarrow K < \frac{N^2}{4(N-1)}. \tag{4}$$

If a CCP does not post collateral to its counterparties, this will lead to an average increase in the counterparties' expected net needs for collateral. Thus, the comparison (4) overstates the benefits of a CCP from the viewpoint of collateral efficiency.

Based on (4), if there are $K = 2$ symmetric classes of derivatives, then central clearing of one of the classes improves netting efficiency if and only if there are at least 7 dealers clearing. If there are 4 symmetric classes of derivatives, then central clearing of one of the classes improves efficiency if and only if there are at least 15 dealers clearing. A CCP is always preferred, in terms of netting efficiency, if it handles all classes of derivatives (which is, in effect, the case of $K = 1$).

It could be argued that the exposure of a dealer to a CCP is likely to be of less concern than its exposure to another dealer, because a CCP is likely to be extremely well capitalized, bearing in mind the systemic risk posed by the potential failure of a CCP. We have not considered this "benefit" of a CCP; our average expected exposure measure weights all counterparty exposures equally. Arguing the other way, the centrality of a CCP implies that its failure risk is far more toxic than that of dealers.[5] Likewise, we do not consider this effect.

Because the failure of a CCP would be relatively catastrophic to the financial system, CCPs may in practice require more collateral for a given exposure than would typically be required for dealer-to-dealer exposures. In terms of collateral use, because we take an equal-weighted approach, our analysis could therefore be biased in favor of

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[5]Examples of clearing house failures include those of Caisse de Liquidation, Paris, (1974), the Kuala Lumpur Commodity Clearing House (1983), and the Hong Kong Futures Guarantee Corporation (1987). See Hills, Rule, Parkinson and Young (1999).

the introduction of CCPs.

Our measure of netting efficiency is based on the total of the expected exposures of a dealer to its counterparties. This measure does not consider concentration risk. Even putting aside the systemic risk of a CCP caused by its centrality, a CCP tends to represent a concentration of exposure to its dealer counterparties. In our simple setting, this is true whenever the number of dealers clearing one of the classes of derivatives is greater than the number of derivatives classes, that is $N > K$. Specifically, the expected exposure of a dealer to its CCP, as a multiple of that dealer's expected exposure to each of its dealer counterparties, is $\sqrt{(N-1)/(K-1)}$. For instance, if there are $N = 10$ dealers and $K = 5$ classes of equally risky derivatives, then after novation of positions in one class to a CCP, the expected exposure of a dealer to the CCP is 50% more than its exposure to any one dealer counterparty. When comparing instead to the expected exposure to a dealer counterparty that existed *before* novation to a CCP, this concentration ratio is $\sqrt{(N-1)/K}$, which is 1.34 in our example. This represents a 34% increase in concentration due to "clearing," under our simple assumptions. For $N = 20$ dealers and $K = 5$ classes of derivatives, the corresponding increase in concentration is 94%.

In general, any success of a CCP in reducing average expected exposures is accompanied by an increase in concentration risk that should be carefully mitigated.

Derivatives Classes with Different Degrees of Risk

We now generalize by considering the netting efficiency allowed by the central clearing of a class of derivatives that may have particularly large exposures, relative to other classes of derivatives. That is, we now allow the expected exposure $E[\max(X_{ij}^k, 0)]$ of class k to be different than that of another class m. Our other assumptions are maintained. A class could include derivatives with more than one underlying asset type. For example, we could group together into a single class for clearing purposes all CDS and all interest-rate swaps.

Suppose that derivatives in Class K are under consideration for clearing. The ratio of a dealer's expected exposure with a given dealer counterparty in this asset class to the total expected exposure with the same counterparty in all other classes combined is

$$R = \frac{E\left[\max\left(X_{ij}^K, 0\right)\right]}{E\left[\max\left(\sum_{k<K} X_{ij}^k, 0\right)\right]}.$$

For example, if all classes have equal expected exposures, then $R = 1/\sqrt{K-1}$, using the fact that expected exposures are proportional to standard deviations. If Class-K exposures are twice as big (in terms of expected exposure) as each of the

other $K - 1$ classes, then $R = 2\sqrt{1/(K-1)}$. A calculation analogous to that shown for the case of asset classes with equal expected exposures leads to the following result.

PROPOSITION 1 *The introduction of a CCP for a particular class of derivatives leads to a reduction in average total expected dealer exposure if and only if*

$$R > \frac{2\sqrt{N-1}}{N-2}, \tag{5}$$

where R is the ratio of the pre-CCP expected dealer-to-dealer exposures of the class in question to the expected dealer-to-dealer exposure of all other classes combined.

For example, we can take the case of $N = 8$ dealers, the number of dealers that are currently working with ICE to create a CCP for clearing their CDS exposures.[6] Under our assumptions, with $N = 8$, clearing the derivatives in a particular class through a CCP improves netting efficiency if and only if the fraction R of a dealer's expected exposure attributable to this class is at least $R = 88\%$ of the total expected exposure of all remaining bilaterally netted classes derivatives. With $N = 26$, the cutoff level drops to $R = 41.7\%$. Although the CDS market poses a large amount of exposure risk, with a total notional market size of roughly \$28 trillion, it would be difficult to make the case that it represents as much as 41.7% of dealer expected exposures in all other "uncleared" derivatives classes combined.

A CCP for CDS and Interest-Rate Swaps?

Gyntelberg and Mallo (2008) provide data on exposures of dealers in OTC markets, in several major asset classes. The latest available data, for June 2008, are shown in Table 1. Although these data merely show gross current credit exposures, and therefore do not incorporate the add-on exposure implications of risky marks to market, these data do give a rough indication of the relative amount of exposure in each of the major underlying asset classes, before netting and collateral. The effect of bilateral netting reduced the total gross exposures shown in Table 1 from \$20.4 trillion to \$3.9 trillion, but because of the manner in which these data are collected, the net exposures do not include the effects of credit default swaps.

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[6]The originally announced dealers are Bank of America, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, J.P. Morgan, Merrill Lynch, Morgan Stanley and UBS. See the press release "IntercontinentalExchange, The Clearing Corporation and Nine Major Dealers Announce New Developments in Global CDS Clearing Solution," October 30, 2008 (available for download at http://files.shareholder.com/downloads/ICE/552548601x0x245132/c396b18c-0e6e-441f-9669-54166e407aaf/344066.pdf). Presumably the acquisition of Merrill Lynch by Bank of America now implies the effect of only $N = 8$.

Table 1: Gross credit exposures of dealers in OTC derivatives markets by asset class, counterparty type, and single versus multi-name CDS, as of June 2008. Source: BIS.

Asset class	Exposure ($ billions)
CDS	3,172
Commodity	2,209
Equity Linked	1,146
Interest Rate	9,263
Foreign Exchange	2,262
Unallocated	2,301
Total	20,353
Total after netting	3,859
CDS by Counterparty	
Dealer to dealer	1,678
Dealer to other financial institution	1,430
Dealer to non-financial customers	65
Total	3,172
CDS by type	
Single name	1,889
Multi-name	1,283
Total	3,172

In light of Proposition 1, it would be hard to base a case for the netting benefits of a central clearing counterparty dedicated to credit default swaps on the magnitudes of OTC derivatives credit exposures shown in Table 1. Credit derivatives account for only about 16% of the total gross exposures. If one assumes that total counterparty expected exposures of a given dealer are proportional, class by class, to the gross credit exposures shown in Table 1, and that X_{ij}^k are independent across k, the implied ratio R of expected exposures on credit derivatives to expected exposures on the total of other classes would be about 32%. This would in turn imply, from Proposition 1, that a central clearing counterparty dedicated to CDS reduces average expected counterparty exposures if there are more than about 36 dealers clearing together. After adding to gross exposures the add-on effect of highly volatile CDS marks to market (relative to other asset classes), the threshold number of dealers necessary to justify a central clearing counterparty dedicated to CDS is likely to be somewhat lower.

Exposures on credit derivatives among dealers have been reduced significantly since

June 2008 due to CDS compression trades.[7]

According to DTCC DerivServ data, dealer CDS positions continue to shrink at this writing. The total size of the CDS market, in terms of notional positions, is now roughly half of mid-2008 levels.

The data suggest that there would be a much stronger case for the joint clearing of CDS and interest-rate swaps, which together accounted for about 60.7% of the total gross exposures. Indeed, interest-rate swaps on their own represent large enough exposures to justify a dedicated central clearing counterparty, and a significant fraction of interest-rate swaps are already cleared through CCPs.[8]

One could argue that CDS exposure is rather special, because of jump-to-default risk and because default risk tends to be correlated with systemic risk. Given the typical practice of daily re-collateralization, the revaluation of CDS positions caused by any defaults on a given day would need to be extremely large in order to build a strong case for separate CDS clearing on the implications of jump-to-default risk. Our results show that jump-to-default risk is better reduced through bilateral netting unless this risk is large relative to that of all other OTC derivatives exposures. Of the total of $3,172 billion in gross credit exposures shown in Table 1 for credit derivatives, $1,283 are on multi-name CDS products, mainly in the form of index contracts such as CDX and iTraxx, which represent equal-weighted CDS positions in over 100 corporate borrowers. These products have relatively small jump-to-default risk, in comparison with single-name CDS.

Initially, at least, a CCP dedicated to CDS clearing would likely restrict its attention to a subset of actively traded CDS. For example, LCH.Clearnet has begun clearing index CDS contracts. The smaller the subset of CDS that is cleared, the lower is the netting efficiency offered by central clearing relative to bilateral netting.

Although we do not study the implications of concentrations of expected exposures among a small subset of dealers, presumably this reduces the benefits of a CCP, because the benefit of multi-lateral netting among a large set of dealers is reduced by concentration. For example, among U.S. banks, data available through the Office of the Comptroller of the Currency show that well over 90% of 2007 CDS volumes were handled by Citigroup, J.P. Morgan, and Bank of America. (These data apply to the

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[7]According to a press release by Markit of July 2, 2008, a compression trade "involves terminating existing trades and replacing them with a far fewer number of new 'replacement trades' which have the same risk profile and cash flows as the initial portfolio, but with less capital exposure. The initiative, available to both the U.S. and European CDS markets, will be managed jointly by Creditex and Markit and has the support of 13 major CDS market participants." See "Markit and Creditex Announce Launch of Innovative Trade Compression Platform to Reduce Operational Risk in CDS Market," July 2, 2008, at www.markit.com.

[8]SwapClear, associated with LCH.Clearnet, is a significant CCP for interest-rate swaps. U.S.-based CCPs for interest-rate swaps are recent, and include CME Cleared Swaps and IDGC. Ledrut and Upper (2007) provide details on the central clearing of interest rate swaps.

period before Goldman Sachs and Morgan Stanley became bank holding companies.) Supposing that Goldman Sachs and Morgan Stanley have dealer businesses in CDS trading that are comparable in size to those of these three banks, the effective number of U.S. CDS dealers for purposes of our analysis may not be much more than 5, given the relatively low levels of CDS positions among other U.S. banks. The proposal for derivatives clearing becomes relatively more attractive if a single CCP handles clearing for all large global dealers, including those in Europe and the U.S, and much more attractive if credit derivatives and cleared together with interest-rate swaps in the same central clearing counterparty.

Cross-Class Exposure Correlation

We now allow for the possibility that derivatives exposures are correlated across asset classes. For simplicity, we suppose that the correlation ρ between X_{ij}^k and X_{ij}^m does not depend on i, j, or the particular pair (k, m) of asset classes. (We continue to assume joint normality, symmetry, and equal variances.) We have already considered the case of $\rho = 0$.

For dealer-to-dealer exposures, it would be reasonable to assume that ρ is small in magnitude, bearing in mind that this correlation depends in part on whether the exposure between i and j in one particular derivative contract is likely to be of the same sign as that of its exposure in another. For pairs of dealers with large matched-book operations, one might anticipate that ρ is close to zero.

The average total expected exposure without a CCP is

$$\phi_{N,K} = \frac{1}{\sqrt{2\pi}}\sigma(N-1)\sqrt{K(1+(K-1)\rho)}. \tag{6}$$

With a CCP for Class-K positions only, the average total expected exposure is

$$\gamma_N + \phi_{N,K-1} = \frac{1}{\sqrt{2\pi}}\sigma\left(\sqrt{N-1} + (N-1)\sqrt{(K-1)(1+(K-2)\rho)}\right). \tag{7}$$

The reduction in average expected exposure due to the introduction of a CCP for one class of derivatives is therefore

$$\theta(N,K) = \phi_{N,K} - (\gamma_N + \phi_{N,K-1}).$$

PROPOSITION 2 *The introduction of a CCP for one class of derivatives reduces the average total expected exposure of a dealer if and only of*

$$\theta(N,K) > 0 \Leftrightarrow \beta_K > \frac{1}{\sqrt{N-1}}, \tag{8}$$

where

$$\beta_K = \frac{1 + 2\rho(K-1)}{\sqrt{K(1 + (K-1)\rho)} + \sqrt{(K-1)(1 + (K-2)\rho)}}.$$

This result follows from the fact that

$$\theta(N,K) = \frac{1}{\sqrt{2\pi}}\sigma(N-1)\left(\sqrt{K(1 + (K-1)\rho)} - \sqrt{(K-1)(1 + (K-2)\rho)} - \frac{1}{\sqrt{N-1}}\right).$$

Rearranging terms, we have the result.

Figure 1 shows the mean reduction in average total expected exposure for various combinations of N, K and ρ. (The reduction is scaled for $\sigma = 1$.) Increasing the correlation between positions increases the relative netting benefits of a CCP, because between-dealer netting is not as beneficial if cross-class exposures are positively correlated.[9]

Because dealers may have a tendency, especially when their counterparties are distressed, of entering derivatives trades that offset exposures arising in other classes of derivatives, there may be some reason to place extra emphasis on the case of negative ρ.

We calculate, treating N as though a real number, that

$$\frac{\partial \theta(N,K)}{\partial N} = \frac{1}{\sqrt{2\pi}}\sigma\left(\beta_K - \frac{1}{2\sqrt{N-1}}\right)$$

$$\frac{\partial^2 \theta(N,K)}{\partial N^2} = \frac{\sigma}{\sqrt{2\pi}}\frac{1}{4(N-1)^{3/2}} > 0.$$

The convexity of $\theta(N,K)$ with respect to N is evident from Figure 1.

Two Central Clearing Counterparties?

We now consider the relative cost or benefit of having two CCPs, as opposed to one, for the same class of derivatives. We return to our original assumption of independence of exposures across classes of exposures. We assume that the dealers are partitioned into two groups for separate clearing, Group A with M dealers and Group B with $N - M$ dealers. We allow for the possibility that dealers within a group have higher exposures with each other than they do with dealers in the other group. Specifically, if dealers i

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[9]For a fixed number N of dealers, as the number K of derivatives classes gets large, β_K converges to $\sqrt{\rho}$, for $\rho > 0$. Thus, in this sense of increasingly many classes of derivatives, or more generally as the expected exposure in the class to be centrally cleared becomes small relative to that in other classes of derivatives, a CCP is asymptotically efficient if and only if $\rho > 1/(N-1)$.
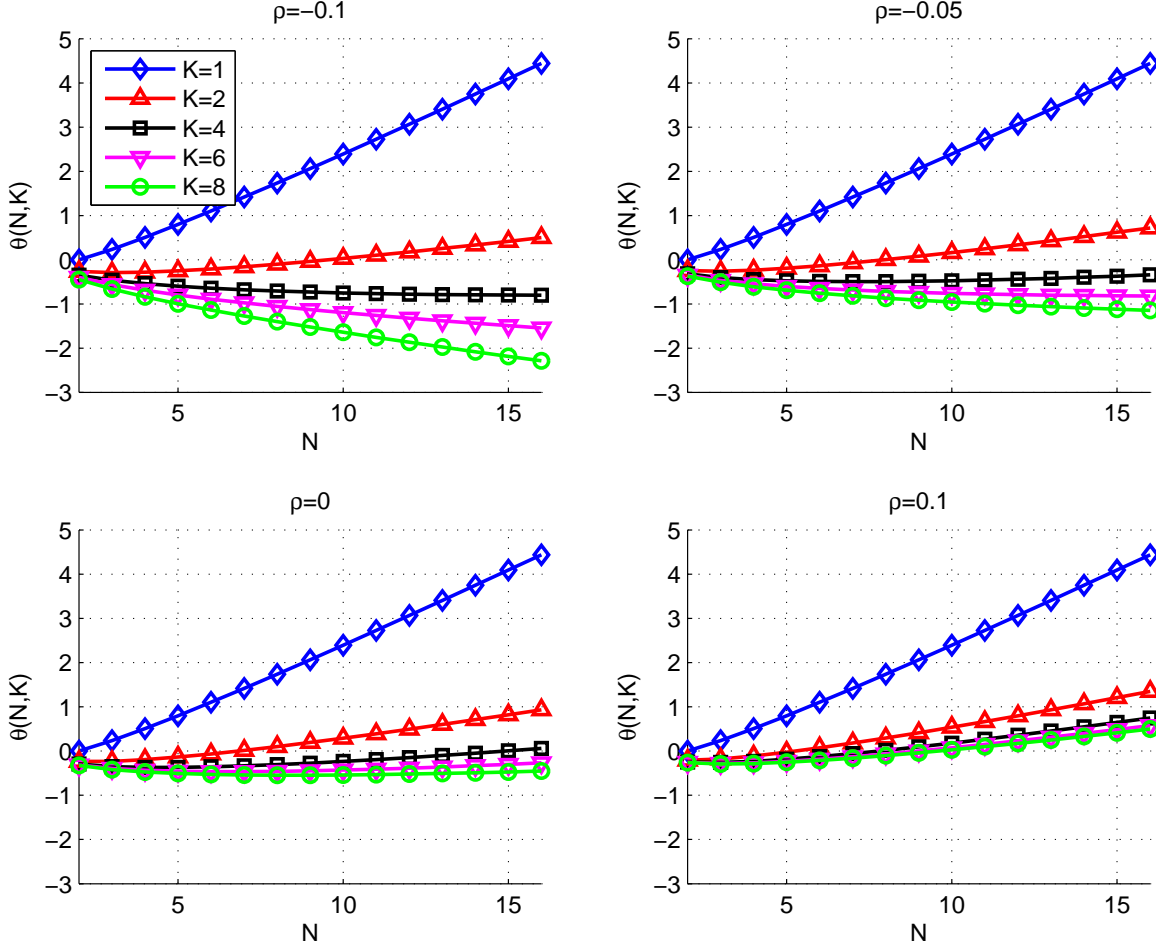
Figure 1: Reduction in average expected exposures associated with clearing one class of derivatives with a single central clearing counterparty, based on N dealers, K classes of derivatives, and a cross-class exposure correlation of ρ. The reductions are scaled for $\sigma = 1$.

and j are in different groups, while i and n are in the same group, we let

$$q = \frac{E[\max(X_{ij}^k, 0)]}{E[\max(X_{in}^k, 0)]}$$

be the ratio of cross-group expected exposures to within-group expected exposures. We will always assume, naturally, that $q \leq 1$. Our assumptions are otherwise as before.

With the introduction of CCPs for Class-K derivatives, one for each group, we suppose that all dealers continue to bilaterally net exposures on the remaining $K - 1$ classes, that they clear Class-K derivatives within their own group, and that they continue to bilterally net exposures on Class-K derivatives with those counterparties that are not in their own group. The total expected exposure of a dealer in Group A,

for instance, is therefore

$$\phi_{M,K-1} + q\phi_{N-M+1,K} + \gamma_M = \frac{1}{\sqrt{2\pi}}\sigma\left((M-1)\sqrt{K-1} + q(N-M)\sqrt{K} + \sqrt{M-1}\right).$$

For $M = N/2$, with N even, the average total expected dealer exposure (in both groups) is

$$\frac{1}{2}\left(\phi_{M,K-1} + q\phi_{N-M+1,K} + \gamma_M + \phi_{N-M,K-1} + q\phi_{M+1,K} + \gamma_{N-M}\right)$$

$$= \frac{1}{\sqrt{2\pi}}\sigma\left[\left(\frac{N}{2}-1\right)\sqrt{K-1} + \frac{qN}{2}\sqrt{K} + \sqrt{\frac{N}{2}-1}\right]$$

Similarly, with only one CCP, the average total expected dealer exposure is

$$\frac{1}{\sqrt{2\pi}}\sigma\left[\left(\frac{N(1+q)}{2}-1\right)\sqrt{K-1} + \sqrt{\frac{N(1+q^2)}{2}-1}\right]$$

We let $\Theta(N, K, M)$ be the reduction in expected exposures associated with two CCPs, over using one CCP for the same class of derivatives for all dealers. For the case of $M = N/2$, we calculate that

$$\Theta(N, K, N/2) = \frac{1}{\sqrt{2\pi}}\sigma\left[-\frac{qN}{2(\sqrt{K}+\sqrt{K-1})} - \sqrt{\frac{N}{2}-1} + \sqrt{\frac{N(1+q^2)}{2}-1}\right]. \quad (9)$$

For $M = N/2$, having two CCPs is more efficient than having one CCP if and only if

$$\Theta(N, K, N/2) > 0 \Leftrightarrow \sqrt{K} + \sqrt{K-1} > \frac{1}{q}\left(\sqrt{\frac{N}{2}-1} + \sqrt{\frac{N(1+q^2)}{2}-1}\right). \quad (10)$$

Without any CCP, the expected exposure is

$$\frac{1}{\sqrt{2\pi}}\sigma\left(\frac{N(1+q)}{2}-1\right)\sqrt{K}.$$

Provided $M = N/2$, a unique CCP for all Class-K derivatives reduces average expected exposure, relative to no CCP, by

$$\delta(N, K, q) = \frac{1}{\sqrt{2\pi}}\sigma\left[\left(\frac{N(1+q)}{2}-1\right)(\sqrt{K}-\sqrt{K-1}) - \sqrt{\frac{N(1+q^2)}{2}-1}\right].$$

Having a single CCP for all dealers improves efficiency, relative to having none, if and

only if

$$\delta(N, K, q) > 0 \Leftrightarrow \sqrt{K} + \sqrt{K-1} < \frac{\frac{N(1+q)}{2} - 1}{\sqrt{\frac{N(1+q^2)}{2} - 1}}. \tag{11}$$

Comparing (10) and (11), for equally sized groups of dealers, one can show that whenever introducing a unique CCP for all dealers strictly improves efficiency, it is always more efficient to have one CPP than to have separate CCPs for each group of dealers. This implication can also be observed in Figure 2.
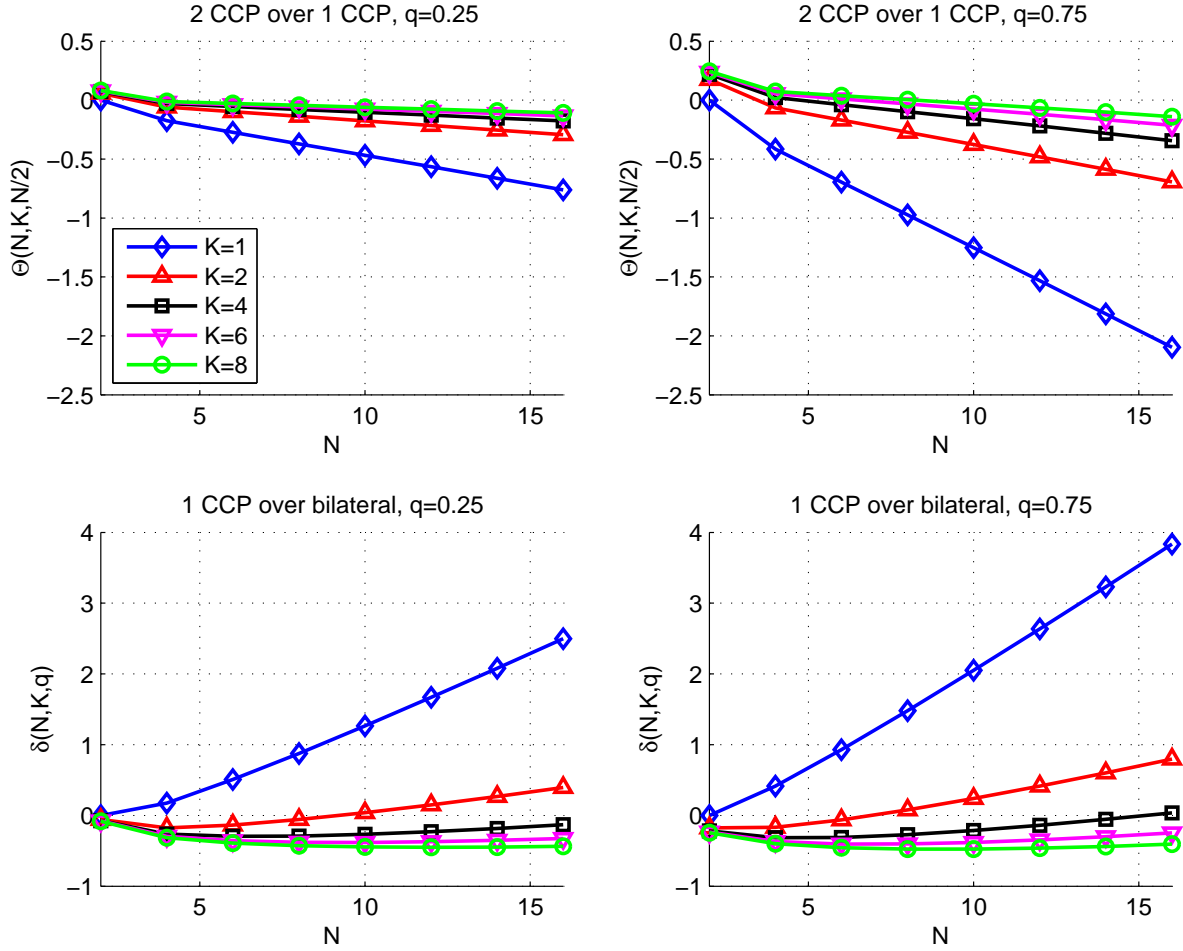

Figure 2: Reductions in average total expected exposure allowed by having two CCPs, one for each group, relative to having one CCP for all dealers, are shown in the top panel. Reductions in average total expected exposure allowed by having one CCP relative to none (fully bilateral netting of exposures) are shown in the bottom panel. The reductions are normalized by taking $\sigma = 1$.

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